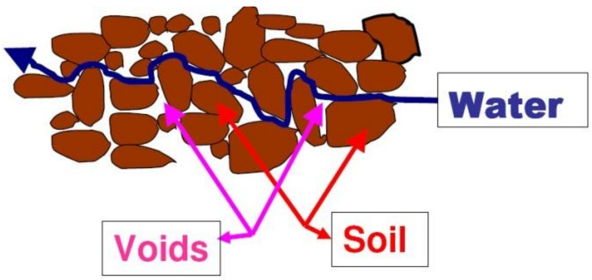
Unit - 3

PERMEABILITY

Flow takes place only when voids are present and they are interconnected. Cork is an example of a material that has voids but these voids are not interconnected. Hence flow cannot take place through a material like cork.

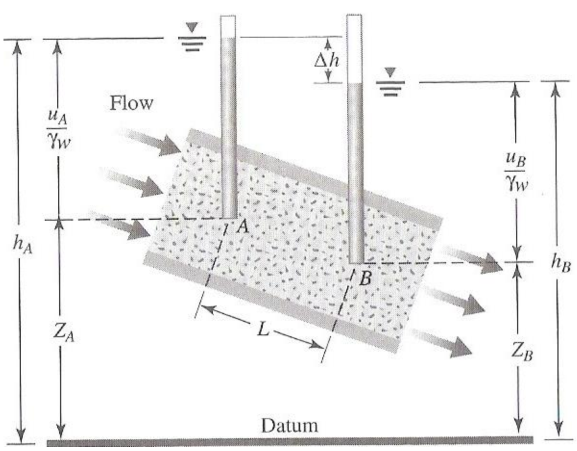


Soils consist of discrete particles, the void spaces between the particles are interconnected and may be viewed as a highly complex and intricate network of irregular tubes. The resistance to flow is greater when the pores are of smaller size and the pore channels irregular. The resistance to flow is much less when the soils have larger voids i.e., more or less regular flow channels. Thus, even while are soils are permeable, the degree of perviousness is different. Gravels are more pervious than sands, sand more pervious than silts and silts more pervious than clays.

Permeability of a soil is the ease with which water flows through that soil. The study of the flow of water through permeable soil media is necessary for

i. estimating the quantity of underground seepage under various hydraulic conditions
ii. investigating problems involving the pumping of water for underground construction
iii. making stability analyses of earth dams and earth retaining structures that are subject to seepage forces
iv. problems involving excavations of open cuts in sands below water table
v. subgrade drainage
vi. rate of consolidation of compressible soils

Pressure, Elevation and Total Heads



According to Bernoulli's equation **Total head** consists of **three components:** elevation head, pressure head and velocity head.

As seepage velocity in soils is normally low, velocity head is ignored, and total head becomes equal to the piezometric head. Due to the low seepage velocity and small size of pores, the flow of water in the pores is steady and laminar in most cases. Water flows from zones of higher potential to zones of lower potential. Thus flow takes place between two points in soil due to the difference in total heads.

Elevation head (***Z***) is the vertical distance of a given point above or below a datum plane.

At point **A,** the pore water pressure ($\boldsymbol{u_A}$) can be measured from the height of water in a standpipe located at that point. The pressure head is the pore water pressure ($\boldsymbol{u_A}$) at that point divided by the unit weight of water, γ_w.

The height of water level in the standpipe above the datum is the **piezometric head (*h*).**

For flow of water through a porous soil medium, velocity head can be neglected because seepage velocity is small.

The total head at any point can be adequately represented by

$$\boldsymbol{h = \frac{u}{\gamma_w} + Z}$$

Figure above shows the relationship among pressure, elevation and total heads for the flow of water through soil. Open standpipes called *piezometers* are installed at points A and B. The levels to which water rises in the piezometers tubes situated at points A and B are known as piezometric levels of points *A* and *B*, respectively. Pressure head at a point is the height of the vertical column of water in the piezometers installed at that point.

The loss of head between the two points *A* and *B*, can be given by

$$\boldsymbol{\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)}$$

The head loss, Δh, can be expressed in a non-dimensional form as

$$\boldsymbol{i = \frac{\Delta h}{L}}$$

where *i* = hydraulic gradient
L = distance between points *A* and *B* – that is, the length of flow over which the loss of head occurred.

The hydraulic gradient, *i*, is the head lost in flow due to friction per unit length of flow. The potential driving the water flow is the hydraulic gradient between the two points, which is equal to the head drop per unit length. In steady state seepage, the gradient remains constant.

At low velocities, the flow through soils remains laminar. For laminar flow, the velocity *v* bears a linear relationship to the hydraulic gradient, *i*. Thus,

$$\boldsymbol{v \propto i}$$

DARCY'S LAW

The flow of free water (water which moves under the influence of gravity) in soils is governed by Darcy's law. Darcy published a simple equation for the discharge velocity of water through saturated soils, which may be expressed as

$$v = k\,i$$

where *v* = **discharge velocity** or **superficial velocity**, which is the quantity of water flowing in unit time through a unit gross cross sectional area of soil at right angles to the direction of flow

k = **coefficient of permeability** also known as **hydraulic conductivity**

Thus, **Darcy's law** states that there is a linear relationship between flow velocity (**v**) and hydraulic gradient (**i**) for any given saturated soil under steady laminar flow conditions.

If the hydraulic gradient is unity, the coefficient of permeability is equal to the velocity of flow.

Thus, **the coefficient of permeability is defined as the velocity of flow which would occur under unit hydraulic gradient**. It has the dimensions of velocity and is measured in mm/s, m/s or m/day.

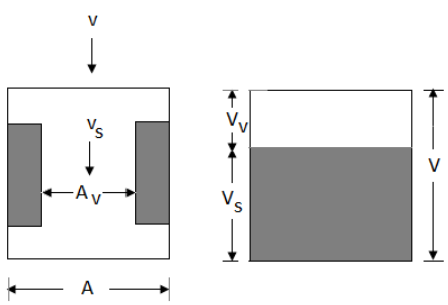
The actual velocity of water (that is the **seepage velocity,** v_s) through the void spaces is greater than *v*.

The discharge *q* is obtained by multiplying the velocity of flow *v* by the gross cross-sectional area (area of both the solids and voids) of soil *A* normal to the direction of flow. Thus,

$$q = kiA$$

RELATIONSHIP BETWEEN SEEPAGE VELOCITY AND DISCHARGE VELOCITY

The total cross-sectional area of soil consists of voids and solids. As the flow can take place only through voids, the actual velocity or seepage velocity (v_s) through the voids is much greater than the discharge velocity (v).



A relationship between the discharge velocity and the seepage velocity can be established referring to the figure, where a longitudinal section through a sample of soil in which the voids and solid particles are segregated.

From the continuity of flow,

$$q = vA = v_s A_v$$

or $$v_s = v \; x \; \frac{A}{A_v}$$

Multiplying the numerator and denominator by the length of the soil sample, *L*,

$$v_s = v\ x\ \left(\frac{AxL}{A_v xL}\right) = \ v\,x\,\frac{V}{V_v}\ =\ \frac{v}{n} = \frac{ki}{n} = \ k_p i$$

where A_V = Area of voids on a cross section normal to the direction of flow, n = porosity and k_p is the **coefficient of percolation.**

The value of coefficient of percolation (k_p) is always greater than the coefficient of permeability(**k**)

PERMEABILITY OF DIFFERENT SOILS

Soil	k (cm/sec)
Gravel	10^0
Coarse sand	10^0 to 10^{-1}
Medium sand	10^{-1} to 10^{-2}
Fine sand	10^{-2} to 10^{-3}
Silty sand	10^{-3} to 10^{-4}
Silt	1 x 10^{-5}
Clay	10^{-7} to 10^{-9}

Permeability (**k**) is an engineering property of soils and is a function of the soil type. Its value depends on the average size of the pores and is related to the distribution of particle sizes, particle shape and soil structure. The ratio of permeabilities of typical sands/gravels to those of typical clays is of the order of $\mathbf{10^6}$. A small proportion of fine material in a coarse-grained soil can lead to a significant reduction in permeability.

For different soil types as per grain size, the orders of magnitude for permeability are as follows:

FACTORS AFFECTING PERMEABILITY

The Poisueille equation for the rate of flow through a tube of any geometrical cross-section is

$$q = \ d_e^2 \frac{\gamma_w}{\eta} \frac{e^3}{1+e} CiA$$

Comparing the above equation with Darcy's law gives

$$k = C d_e^2 \frac{\gamma_w}{\eta} \frac{e^3}{1+e}$$

Where C is composite shape factor dependent on grain shape and d_e is representative grain size.

From the equation above, **the factors affecting the permeability are**

i. **Particle shape and size**
   - Permeability varies with the shape (factor C in the above equation) and size of the soil particles.
   - Permeability varies with the square of particle diameter.
   - Smaller the grain-size the smaller the voids and thus lower the permeability.
   - A relationship between permeability and grain-size is more appropriate in case of sands and silts.
   - Allen Hazen proposed the following empirical equation, $k = \ CD_{10}^2\ cm/s$

C is a constant that varies from 1.0 to 1.5 and D_{10} is the effective size in mm

ii. Void ratio

- For a given soil, greater the void ratio, the higher is the value of coefficient of permeability.
- It causes an increase in the percentage of cross-sectional area available for flow.

iii. Degree of saturation

- Higher the degree of saturation, higher is the permeability.
- In the case of certain sands the permeability may increase three-fold when the degree of saturation increases from 80% to 100%.

iv. Adsorbed water

- Permeability depends on the thickness adsorbed water. Adsorbed water is not free to move under gravity.
- It causes an obstruction to flow of water in the pores and hence reduces the permeability.

v. Soil structure

- Fine-grained soils with a flocculated structure have a higher coefficient of permeability than those with a dispersed structure.
- Remoulding of a natural soil reduces the permeability
- Permeability parallel to stratification is much more than that perpendicular to stratification

vi. Presence of entrapped air and other foreign matter.

- Entrapped air reduces the permeability of a soil.
- Organic foreign matter may choke flow channels thus decreasing the permeability

vii. Properties of permeant (water)

- Coefficient of permeability (k) is directly proportional to unit weight of water γ_w and inversely proportional to its viscosity (η).
- k increases with an increase in temperature due to reduction in the viscosity.

MEASUREMENT OF PERMEABILITY

The coefficient of permeability can be determined in three ways:

- Laboratory tests
- Field tests
- Empirical approach

Laboratory methods

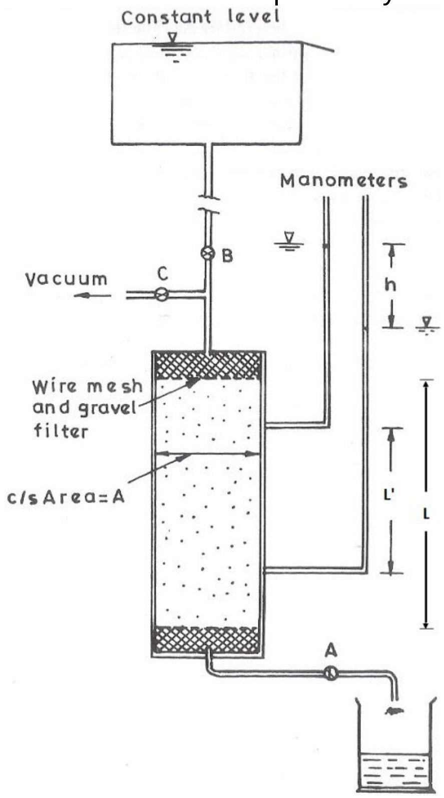
In the laboratory, it is determined using either constant head or variable head test.

Constant Head Test - IS:2720 (Part XVII)

The test is conducted in an instrument known as permeameter. Constant head permeters are specially suited to the testing of pervious, coarse grained soils, since adequate, measurable discharge is needed for the accurate determination of permeability by this method. The permeameter consiss of a metallic mould, 100 mm internal diameter, 127.3 mm effective height and 1000 ml capacity. The sample is placed between two porous discs and the whole assembly is placed in a constant head chamber filled with water to the brim at the start of the test.

A typical set up is shown in figure. Water is allowed to flow through the soil sample from a *constant head reservoir* designed to keep the keep water level constant by overflow. The quantity of water flowing out of the sample in to *the constant head chamber* spills over the chamber and the discharge Q during a given time t is collected in a measuring cylinder.

Since the presence of entrapped air in the soil can affect the permeability, de-aired water is supplied to the reservoir and then vacuum is applied to the soil sample before commencing the test. It is essential that the sample is fully saturated.



If the cross-sectional area of the specimen is *A* and Q is the volume of water collected in time t, the discharge is given by

$$q = kiA$$

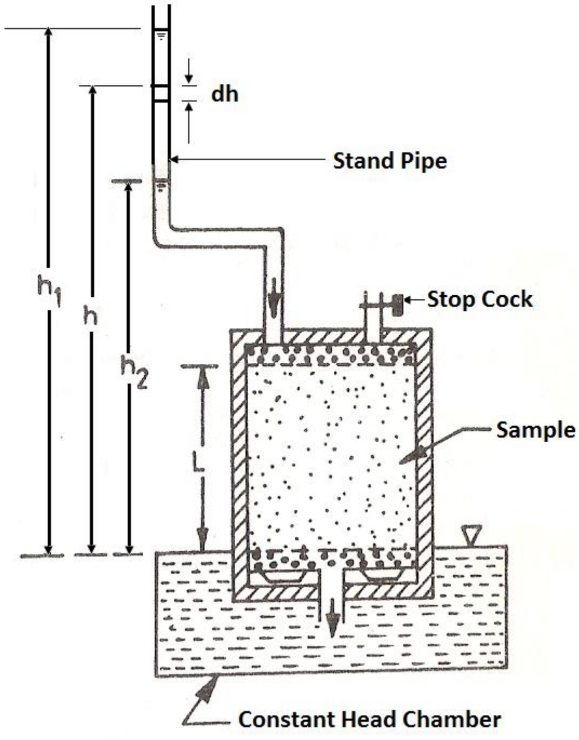
$$q = k\frac{h}{L}A$$

$$k = \frac{qL}{Ah} = \frac{QL}{Aht}$$

Falling Head Test or Variable Head Test

This method is used to determine the permeability of fine-grained soils such as fine sands and silts. In such soils, the permeability is too small to enable accurate measurement of discharge using a constant head permeameter. The permeameter mould is the same as that used in the constant head test. A vertical graduated standpipe of known diameter is fitted to the top of permeameter. The sample is placed between two porous discs and the whole assemble is placed in a constant head chamber filled with water to the brim at the start of the test.

The test is started by allowing water in the standpipe to flow through the sample to the constant head chamber from which it overflows and spills out. As the water flows through the soil, the water level in the standpipe falls. The time required for water level to fall from a known initial head (h_1) to the known final head (h_2) is determined. The head is measured with reference to the level of water in the constant head chamber.



Consider the instant when the head is h. For the infinitesimal small time dt, the head falls by dh. Let the discharge through the sample be q. Let the cross-sectional area of the standpipe be a. From the continuity of flow,

$$\boldsymbol{a.dh = -q.dt}$$

From Darcy's law, $q = kiA$. Substituting for q,

$$\boldsymbol{a.dh = -(A.k.i)dt}$$

$$\boldsymbol{a.dh = -Ak\frac{h}{L}dt}$$

Rearranging the terms,

$$\boldsymbol{\frac{Akdt}{aL} = -\frac{dh}{h}}$$

Integrating,

$$\boldsymbol{\frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2}\frac{dh}{h}}$$

$$\boldsymbol{\frac{Ak}{aL}(t_2 - t_1) = \log_e\frac{h_1}{h_2}}$$

$$\boldsymbol{k = \frac{aL}{At}\log_e\frac{h_1}{h_2}}$$

$$\boldsymbol{k = \frac{2.30aL}{At}\log_{10}\frac{h_1}{h_2}}$$

where $t = (t_2 - t_1)$, the time interval during which the head reduces from h_1 to h_2.

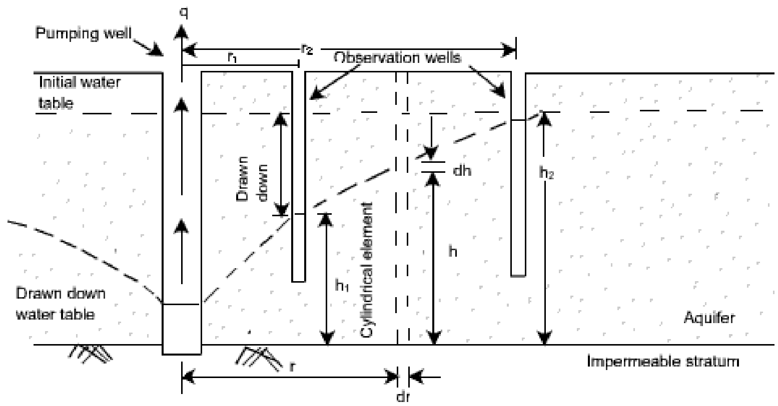
FIELD TESTS FOR PERMEABILITY

Field or *in-situ* measurement of permeability avoids the difficulties involved in obtaining and setting up undisturbed samples in a permeameter. It also provides information about bulk permeability, rather than merely the permeability of a small sample.

A field permeability test consists of pumping out water from a main well and observing the resulting drawdown surface of the original horizontal water table from at least two observation wells. When a steady state of flow is reached, the flow quantity and the levels in the observation wells are noted.

Two important field tests for determining permeability are: Unconfined flow pumping test, and confined flow pumping test.

Unconfined Flow Pumping Test



In this test, the pumping causes a drawdown in an unconfined (i.e. open surface) soil stratum, and generates a radial flow of water towards the pumping well. The steady-state heads $\mathbf{h_1}$ and $\mathbf{h_2}$ in observation wells at radii $\mathbf{r_1}$ and $\mathbf{r_2}$ are monitored till the flow rate **q** becomes steady.

The rate of radial flow through any **cylindrical surface** around the pumping well is equal to the amount of water pumped out. Consider such a surface having radius **r**, thickness **dr** and height **h.** The hydraulic gradient is

$$i = \frac{dh}{dr}$$

Area of flow, *A*, is

$$A = 2\pi r h$$

From Darcy's Law,

$$q = k.i.A$$

$$q = k.\frac{dh}{dr}.2\pi r h$$

Arranging and integrating,

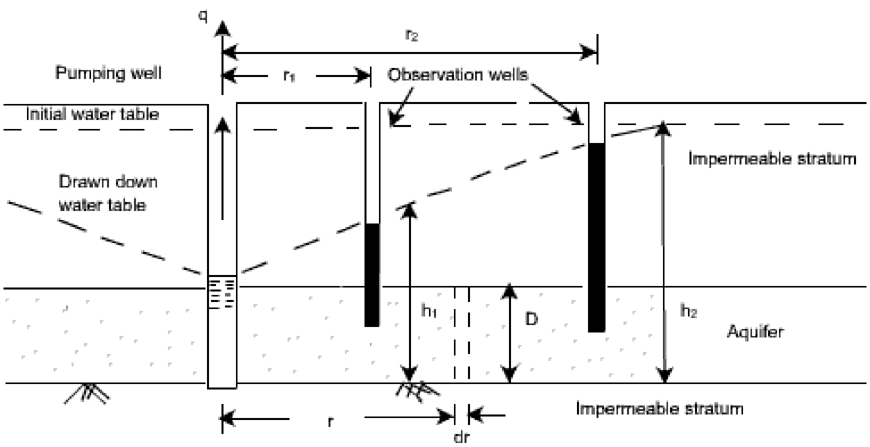
$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{q} \int_{h_1}^{h_2} h dh$$

$$\log_e \frac{r_2}{r_1} = \frac{\pi k}{q}\left(h_2^2 - h_1^2\right)$$

$$k = \frac{q.\log_e \frac{r_2}{r_1}}{\pi\left(h_2^2 - h_1^2\right)}$$

$$k = \frac{2.303q \log_{10} \frac{r_2}{r_1}}{\pi\left(h_2^2 - h_1^2\right)}$$

Confined Flow Pumping Test



Artesian conditions can exist in a aquifer of thickness **D** confined both above and below by impermeable strata. In this, the drawdown water table is above the upper surface of the aquifer.

For a **cylindrical surface** at radius **r**,

$$q = k.\frac{dh}{dr}.2\pi r D$$

Integrating,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k D}{q} \int_{h_1}^{h_2} dh$$

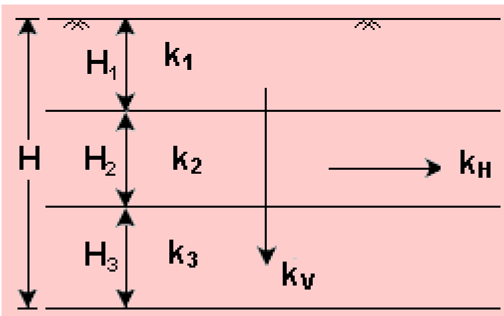
$$\log_e \frac{r_2}{r_1} = \frac{2\pi k D}{q}(h_2 - h_1)$$

$$k = \frac{2.303q \log_{10} \frac{r_2}{r_1}}{2\pi D(h_2 - h_1)}$$

PERMEABILITY OF STRATIFIED SOILS

When a soil deposit consists of a number of horizontal layers having different permeabilities, the average value of permeability can be obtained separately for both vertical flow and horizontal flow, as $\mathbf{k_V}$ and $\mathbf{k_H}$ respectively.

Consider a stratified soil having horizontal layers of thickness $\mathbf{H_1}$, $\mathbf{H_2}$, $\mathbf{H_3}$, etc. with coefficients of permeability $\mathbf{k_1}$, $\mathbf{k_2}$,$\mathbf{k_3}$, etc.



For vertical flow
The flow rate **q** through each layer per unit area is the same.

$$q = q_1 = q_2 = \ldots\ldots\ldots\ldots\ldots$$

Let **i** be the equivalent hydraulic gradient over the total thickness **H** and let the hydraulic gradients in the layers be $\mathbf{i_1}$, $\mathbf{i_2}$, $\mathbf{i_3}$, etc. respectively.

$$k_V.i = k_1.i_1 = k_2.i_2 = \ldots\ldots\ldots\ldots\ldots\ldots$$ where $\mathbf{k_V}$ = Average vertical permeability

$$k_V.\frac{h}{H} = k_1.\frac{h_1}{H_1} = k_2.\frac{h_2}{H_2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

The total head drop **h** across the layers is

$$h = h_1 + h_2 + \ldots\ldots\ldots\ldots\ldots$$

$$h = \frac{k_V.h}{H}.\frac{H_1}{k_1} + \frac{k_V.h}{H}.\frac{H_2}{k_2} + \ldots\ldots\ldots\ldots$$

$$k_V = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots}$$

Horizontal flow
When the flow is horizontal, the hydraulic gradient is the same in each layer, but the quantity of flow is different in each layer.

$$i = i_1 = i_2 = i_3 = \ldots\ldots$$

The total flow is

$$q = q_1 + q_2 + q_3 \ldots\ldots$$

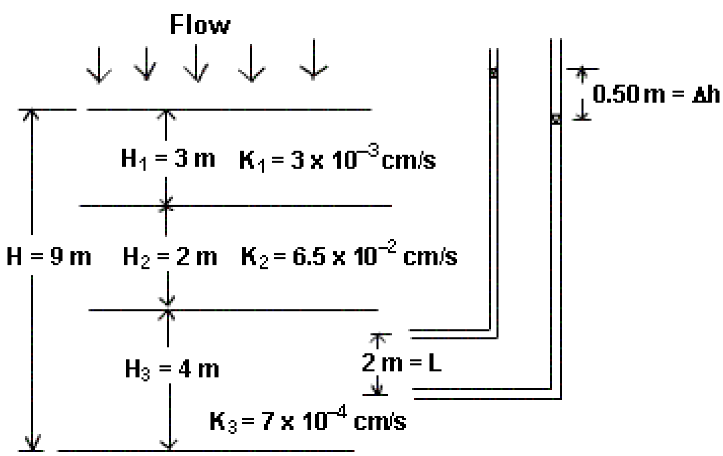
Considering unit width normal to the cross-section plane,

$$k_H i.H = k_1 i_1.H_1 + k_2 i_2.H_2 + \ldots\ldots\ldots\ldots$$

$$k_H = \frac{1}{H}(k_1.H_1 + k_2 H_2 + \ldots\ldots\ldots\ldots)$$

Example 1: Determine the following:

(a) Equivalent coefficient of vertical permeability of the three layers
(b) The rate of flow per m^2 of plan area
(c) The total head loss in the three layers



Solution:

(a) $K_v = \dfrac{H}{\dfrac{H_1}{K_1} + \dfrac{H_2}{K_2} + \dfrac{H_3}{K_3}} = \dfrac{9}{\dfrac{3}{3x10^{-3}} + \dfrac{2}{6.5x10^{-2}} + \dfrac{4}{7x10^{-4}}} = 1.33 \times 10^{-3}$ cm/s

(b) Considering an area A = 1 m^2 =1 x 10^4 cm^2

q = k.i.A = $k_3.\dfrac{\Delta h}{L}.A$ = 7 x 10^{-4} x 0.25/2 x ($1x10^4$) = 0.875 cm^3/s per m^2 of plan area

(c) For continuity of flow, velocity is the same.

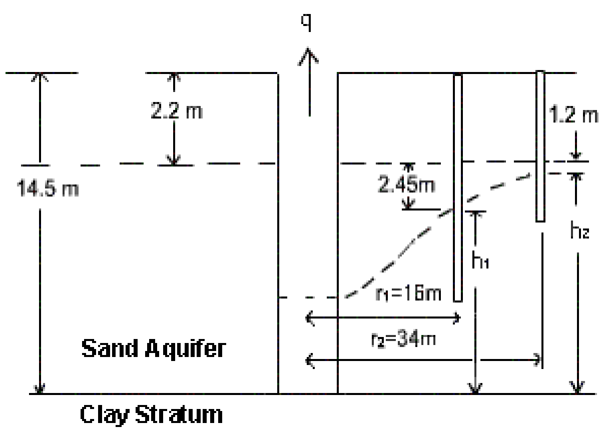
$k_3 \cdot \frac{\Delta h}{L} = k_V \cdot \frac{\Delta h_{total}}{H}$, where Δh_{total} = total head loss in three layers

$\therefore \Delta h_{total} = k_3 \cdot \frac{\Delta h}{L} \cdot \frac{H}{k_V} = 7x10^{-4} x \frac{0.50}{2} x \frac{9}{1.33x10^{-3}}$ = 1.184 m

Example 2: For a field pumping test, a well was sunk through a horizontal stratum of sand 14.5 thick and underlain by a clay stratum. Two observation wells were sunk at horizontal distances of 16 m and 34 m respectively from the pumping well.The initial position of the water table was 2.2 m below ground level.

At a steady-state pumping rate of 1850 litres/min, the drawdowns in the observation wells were found to be 2.45 m and 1.20 m respectively. Calculate the coefficient of permeability of the sand.

Solution:



$$k = \frac{q.\log_e(\frac{r_2}{r_1})}{r(h_2^2 - h_1^2)}$$

q = 1850 litres/min = $\frac{1850x10^{-3}}{60} m^3/s$

r_1 = 16 m
r_2 = 34 m
h_1 = 14.5 - 2.2 - 2.45 = 9.85 m
h_2= 14.5 - 2.2 - 1.2 = 11.1 m

$$k = \frac{\frac{1850 x 10^{-3}}{60} x \log e(\frac{34}{16})}{r[(11.1)^2 - (9.85)^2]} = 2.82 \times 10^{-4} \text{ m/s} = 1.41 \times 10^{-2} \text{ cm/s}$$

SEEPAGE IN SOILS

Seepage is the flow of water under gravitational forces in a permeable medium. The flow of water through soil, in many instances, is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In the kind of seepage that takes place around sheet pile walls, under masonry dams and other water retaining structures and through earth dams, embankments etc., the flow condition is two dimensional. In a two dimensional flow, the velocity components in the horizontal and vertical directions vary from point to point within the cross-section of the soil mass. The three dimensional flow is the most general flow situation but the analysis of such problems is too complex to be practical and, hence, flow situations are simplified to the two dimensional flow.

In two-dimensional flow, ground water flow is generally calculated by the use of graphs referred to as flow nets. The concept of low net is based on Laplace's equation of continuity, which governs the steady flow condition for a given point in the soil mass.

The study of two dimensional flow of water is used to

- Calculate flow under and within earth structures.
- Calculate seepage stresses, porewater pressure distribution, uplift forces, hydraulic gradients, and the critical hydraulic gradient.
- Determine the stability of simple geotechnical systems subjected to two-dimensional flow of water.

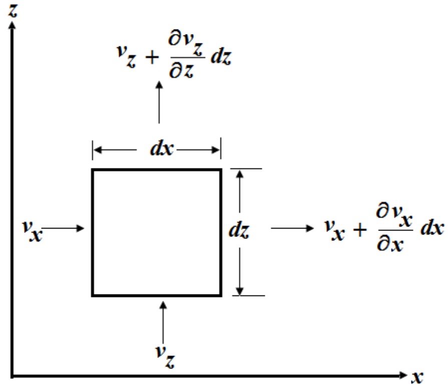
TWO DIMENSIONAL FLOW – LAPLACE'S EQUATION

Assumptions

1. The soil mass is fully saturated.
2. Darcy's law is valid.
3. The soil mass is homogeneous and isotropic.
4. Both the soil grains forming the skeleton and pore fluid are incompressible i.e. no compression or expansion takes place during the flow.
5. Flow conditions do not change with time, that is, steady state conditions exist.

Consider the flow of water into an element in a saturated soil, with dimensions *dx* and *dz* in the horizontal and vertical directions as shown in figure below. The third dimension is along y-axis is very large. For convenience, it is taken as unity.

Let,



Velocity at the inlet face in the horizontal direction = $\boldsymbol{v_x}$

Velocity at the inlet face in the vertical direction = $\boldsymbol{v_z}$

Velocity at the outlet face in the horizontal direction = $\boldsymbol{v_x + \frac{\partial v_x}{\partial x} . dx}$

Velocity at the outlet face in the vertical direction = $\boldsymbol{v_z + \frac{\partial v_z}{\partial z} . dz}$

As the flow is steady and the soil is incompressible, the discharge entering the element is equl to that leaving the element.

Thus,

$$\boldsymbol{v_x dz + v_z dz = \left(v_x + \frac{\partial v_x}{\partial x}.dx\right) dz + \left(v_z + \frac{\partial v_z}{\partial z}.dz\right) dx}$$

$$\boldsymbol{\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) dxdz = 0}$$

As dx.dz ≠ 0,

$$\boldsymbol{\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z}\right) = 0}$$

The above equation is the **continuity equation** for two-dimensional flow.

Let *h* be the total head at any point. The horizontal and vertical components of the hydraulic gradient are

$$\boldsymbol{i_x = -\frac{\partial h}{\partial x} \text{ and } i_z = -\frac{\partial h}{\partial z}}$$

The minus indicates that the head decreases in the direction of flow.

From Darcy's law

$$\boldsymbol{v_x = -k_x \frac{\partial h}{\partial x} \text{ and } v_z = -k_z \frac{\partial h}{\partial z}}$$

Substituting for v_x and v_z, the equation may be written as

$$\boldsymbol{-k_x \frac{\partial^2 h}{\partial x^2} - k_z \frac{\partial^2 h}{\partial z^2} = 0}$$

$$\boldsymbol{k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0}$$

As the soil is isotropic, $k_x = k_z$. Therefore, the continuity equation for two-dimensional flow, known as **Laplace equation** may be written as,

$$\boldsymbol{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$

The Laplace equation can be solved if the boundary conditions at the inlet and exit are known. The equation represents two families of curves which are orthogonal to each other. One family represents the flow lines. ***A flow line is a line along which the water particle will travel from upstream to the downstream side in the permeable soil medium.*** The other family represents the equipotential lines. ***An equipotential line is a line along which the total head at all points is equal.***

FLOW NETS

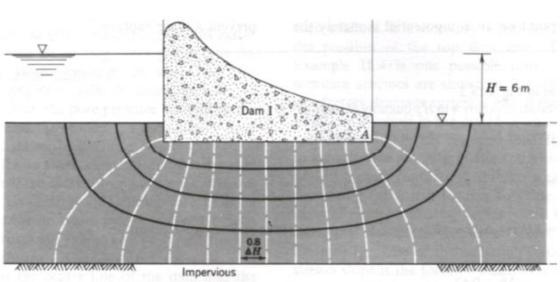
A combination of a number flow lines and equipotential lines is called a **flow net.** A flow net is graphical representation of a flow field (solution of Laplace's equation). Flow nets are constructed for the calculation of groundwater flow and the evaluation of heads in the permeable soil media.

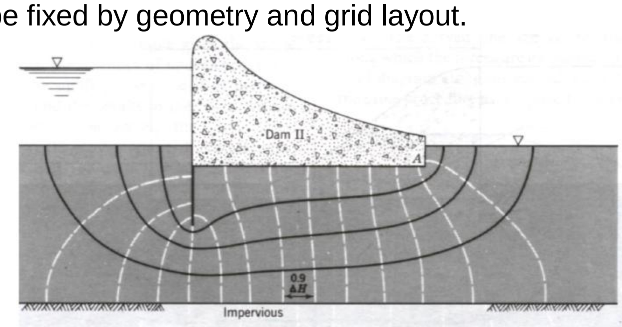
CHARACTERISTICS OF FLOW NETS

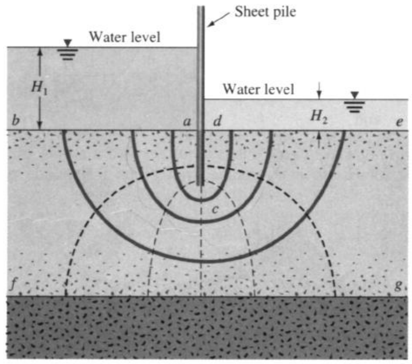
1. Flow lines or stream lines represent flow paths of particles of water
2. Flow lines and equipotential line are orthogonal to each other
3. The area between two flow lines is called a flow channel
4. The rate of flow in a flow channel is constant (Δq)
5. Flow cannot occur across flow lines
6. An equipotential line is a line joining points with the same head
7. The velocity of flow is normal to the equipotential line
8. The difference in head between two equipotential lines is called the potential drop or head loss) (Δh)
9. A flow line cannot intersect another flow line.
10. An equipotential line cannot intersect another equipotential line
11. There can be no flow along an equipotential line as there is no hydraulic gradient.

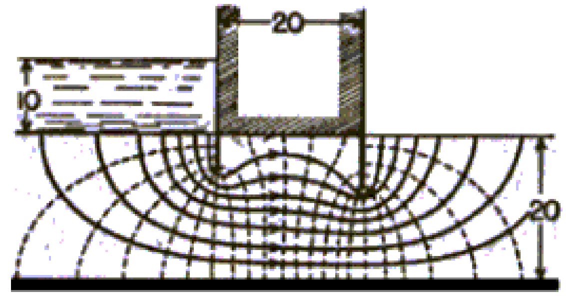
GUIDELINES FOR DRAWING FLOW NETS

- Draw the cross section of the structure, soil mass and water elevations to a suitable scale.
- Identify the equipotential and flow boundaries. ***The soil and impermeable boundary interfaces are flow lines. The soil and permeable boundary interfaces are equipotential lines.***
- Draw a few flow lines and equipotential lines. Sketch intermediate flow lines and equipotential lines by smooth curves adhering to right-angle intersections such that area between a pair of flow lines and a pair of equipotential lines is approximately a curvilinear square grid.
- Where flow direction is a straight line, flow lines are equal distance apart and parallel. Also, the flownet in confined areas between parallel boundaries usually consists of flow lines and equipotential lines that are elliptical in shape and symmetrical
- Try to avoid making sharp transition between straight and curved sections of flow and equipotential lines. Transitions must be gradual and smooth. Continue sketching until a problem develops.
- Successive trials will result in a reasonably consistent flow net. In most cases, 3 to 8 flow lines are usually sufficient. Depending on the number of flow lines selected, the number of equipotential lines will automatically be fixed by geometry and grid layout.





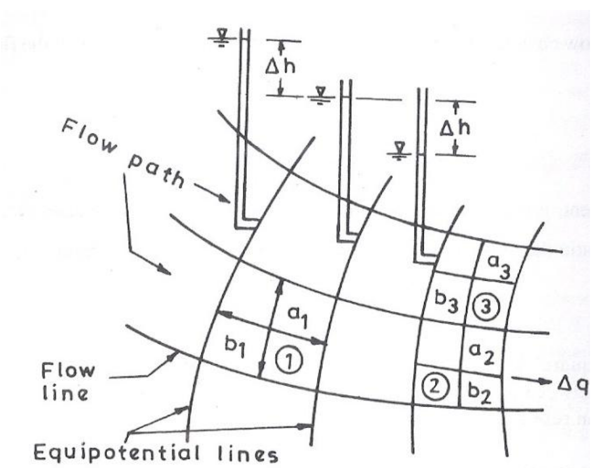




USES OF FLOW NETS

Head loss and seepage calculation

Refer to fig shown below. Consider two fields 1 and 2 lying between two successive flow lines. The average dimensions of the two fields along and normal to the flow are b_1, a_1 and b_2, a_2, respectively. Consider a third field 3 in such a way that fields 2 and 3 lie within the same equipotential lines. The dimensions of field 3 are b_3, a_3.



Let Δq_1, Δq_2, and Δq_3 be the rate of flow through fields 1 , 2 and 3, respectively, and Δh_1, Δh_2, and Δh_3 be the head loss across the fields 1, 2 and 3.

Consider the flow per unit width perpendicular to the plane of the section. From Darcy's law,

$$\Delta q_1 = k\frac{\Delta h_1}{b_1}(a_1\ x\ 1)$$

$$\Delta q_2 = k\frac{\Delta h_2}{b_2}(a_2\ x\ 1)$$

$$\Delta q_3 = k\frac{\Delta h_3}{b_3}(a_3\ x\ 1)$$

Fields 1 and 2 are within the same flow channel. Hence $\Delta q_1 = \Delta q_2$, since there can be no flow across the flow lines.

Since fields 2 and 3 are within the same equipotential lines, , $\Delta h_2 = \Delta h_3$. If all the fields are elementary squares,

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = 1$$

Thus,

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \Delta q$$

$$\Delta h_1 = \Delta h_2 = \Delta h_3 = \Delta h$$

Thus, in a flow net constructed such that all its fields are elementary squares, the quantity of flow through each field will be equal and the head loss across each field will also be equal.

Thus,

$$\Delta q = k.\Delta h$$

If H is the total head loss during the flow, and n_d = number of equipotential drops, then

$$\Delta h = \frac{H}{n_d}$$

$$\therefore \ \Delta q = k\frac{H}{n_d}$$

If the number of flow channels in a flow net is n_f, the rate of flow q through all the flow channels per unit length is

$$q = \Delta q \ x \ n_f$$

$$q = kH\frac{n_f}{n_d}$$

It is not necessary to make all fields elementary squares in flow net. It is only necessary to make $\frac{a}{b}$ ratio the same for all the fields. If $\frac{a}{b} = n$

$$q = kH\frac{n_f}{n_d}n$$

Maximum hydraulic gradient or exit hydraulic gradient and piping

An unstable condition arises when seepage lines emerge vertically upwards on the downstream boundary. The hydraulic gradient of flow will be maximum adjacent to the toe of a dam, and at the base for a sheet pile because it is here that the curvilinear squares are th smallest in size and thus the length of flow for a certain constant drop in head will be smallest.

The maximum hydraulic gradient is $i_{max} = \frac{\Delta h}{L_{min}}$

L_{min} = minimum length of an equipotential drop. Usually, L_{min} occurs at exits. .

A factor of safety of at least 6 is recommended for safety against piping.

Piping Effects

Soils can be eroded by flowing water. Erosion can occur underground, beneath the hydraulic structures, if there are cavities, cracks in rock, or high exit gradient induced instability at toe of the dam, such that soil particles can be washed into them and transported away by high velocity seeping water. This type of underground erosion progresses and creates an open path for flow of water; it is called “piping”. Preventing piping is a prime consideration in the design of safe dams. Briefly the processes associated with initiation of piping in dams are as follows,

- Upward seepage at the toe of the dam on the downstream side causes local instability of soil in that region leading erosion.
- A process of gradual erosion and undermining of the dam may begin, this type of failure known as piping, has been a common cause for the total failure of earth dams
- The initiation of piping starts when exit hydraulic gradient of upward flow is close to critical hydraulic gradient

Factor of safety against piping is defined as,

$$FS = \frac{i_{critical}}{i_{exit}}$$

Where i_{exit} is the maximum exit gradient and $i_{critical}$ is the critical hydraulic gradient. Exit gradient must never come close to the critical hydraulic gradient. The maximum exit gradient can be determined from the flow net. A factor of safety of at least 6 is considered adequate for the safe performance of the structure against piping failure. Exit gradient can be reduced to a considerable extent by providing vertical cut off walls at the downstream end of the base of the dam.

Porewater pressure

The porewater pressure at a given point is calculated by multiplying the pressure head at that point with the unit weight of water.

Uplift pressure

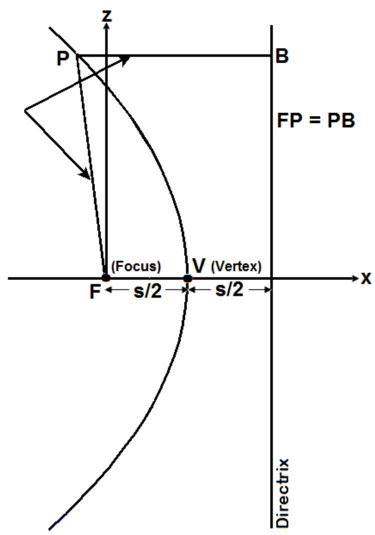
Uplift pressure at any given point is the pore water pressure acting vertically upward due to residual pressure head at that point. The uplift force is calculated by working out the area of the uplift pressure distribution diagram. The total uplift force acts opposite the force of gravity due to the weight of the dam and hence reduces the stability of the hydraulic structure. Uplift pressures along the base of masonry dams can be reduced by providing vertical cut off walls at the upstream end of the base of the dam.

Seepage Flow through Homogeneous Earth Dams

Flow through earth dams is an important design consideration. In order to ensure that the pore water pressure at the downstream end does not lead to instability and the exit hydraulic gradient does not lead to piping, the flow net must be drawn and analysed.

In the case of earth dams, the top boundary flow line, which is a free water surface, is not evident from the geometrical boundaries and therefore, the flow space is not fully defined. The major exercise is to locate the position of the top flow line of seepage in the cross section. The top flow line is called the phreatic line. The pressure on the phreatic line is zero.

Casagrande showed that the phreatic surface can be approximated by a parabola with corrections at the points of entry and exit. The assumed parabola representing the uncorrected phreatic line is called the basic parabola. The properties of the regular parabola which are essential to obtain phreatic line are depicted in figure below.



Every point on the parabola is equidistant from the focus and directrix.

Therefore, **FP = PB**

Also, the distance between the focus and vertex, and vertex and directrix is equal.

Therefore, **FV = p =s/2**

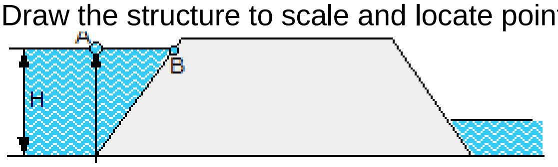
For a point P(x, z),

$$x^2 + z^2 = (2p + x)^2$$

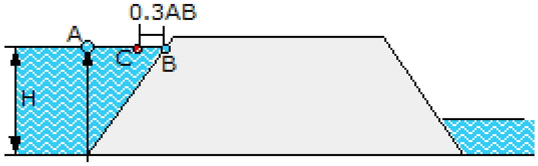
$$x = \frac{z^2 - 4p^2}{4p}$$

PHREATIC LINE FOR AN EARTH DAM WITHOUT TOE FILTER

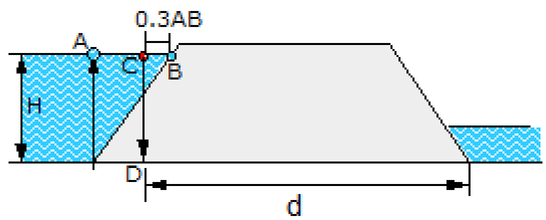
1. Draw the structure to scale and locate points A and B as shown



2. Locate point C such that BC = 0.3AB



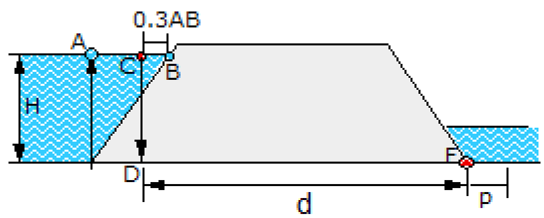
3. Project a vertical line from C as shown



4. Locate the focus, F, at the toe of the dam and calculate the focal distance p as

$$p = \frac{\sqrt{d^2 + H^2} - d}{2}$$

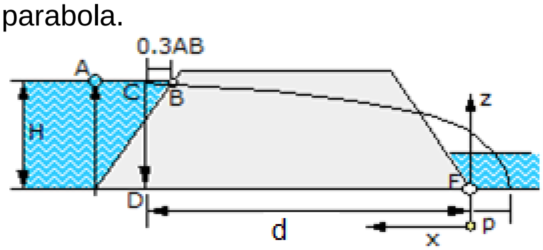
The directrix is located at a distance 2p from the focus F.



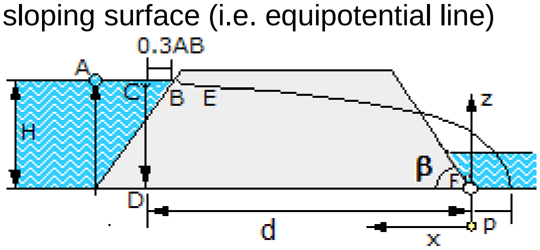
5. Construct basic parabola from

$$z = 2\sqrt{p(p + x)}$$

Choose arbitrary values of x and compute z. Join all the points to get the basic parabola.



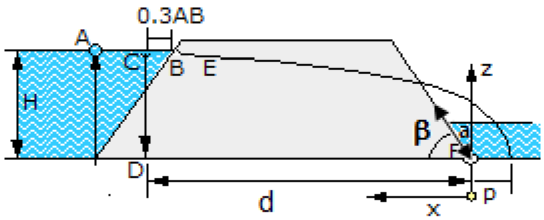
6. Upstream Correction: Sketch the section BE such that it is normal to the upstream sloping surface (i.e. equipotential line)



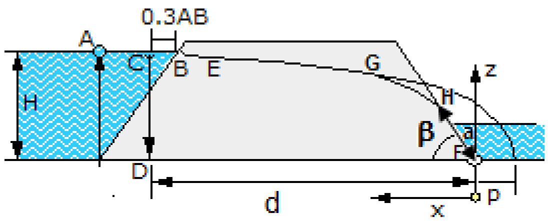
7. Calculate the distance 'a' from

$$for\ \beta \leq 30^o, \qquad a = \frac{d}{\cos\beta} - \sqrt{\frac{d^2}{\cos^2\beta} - \frac{H^2}{\sin^2\beta}}$$

$$for\ 30^o < \beta < 60^o, \qquad a = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 cot^2\beta}$$



8. Downstream Correction: Sketch in a transition section GH



The curve BEGHF is the phreatic line for an earth dam without toe filter.

9. The discharge can be computed as

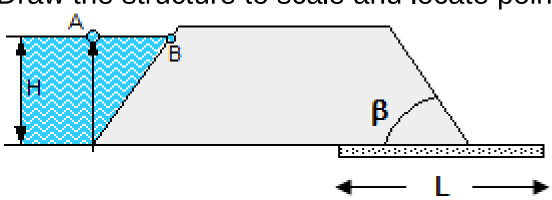
$$for\ \beta < 30^o, \qquad q = k(a\ sin\beta)tan\beta$$

$$for\ 30^o < \beta < 60^o, \qquad q = k\ asin^2\beta$$

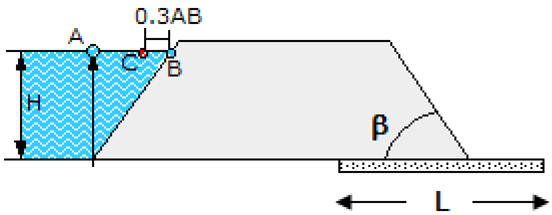
Where k = coefficient of permeability of the earth dam material.

PHREATIC LINE FOR AN EARTH DAM WITH TOE FILTER

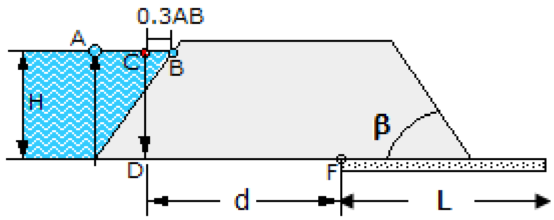
1. Draw the structure to scale and locate points A and B as shown.



2. Locate point C such that BC = 0.3AB

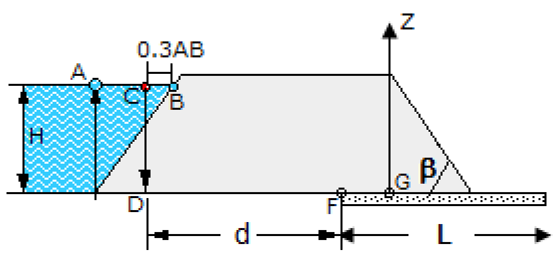


3. Project a vertical line from C as shown. Select F as the focus of the parabolic phreatic line.



4. Locate point G on the directrix at a distance 2p from the focal point.

$$\boldsymbol{p = \frac{\sqrt{d^2 + H^2} - d}{2}}$$

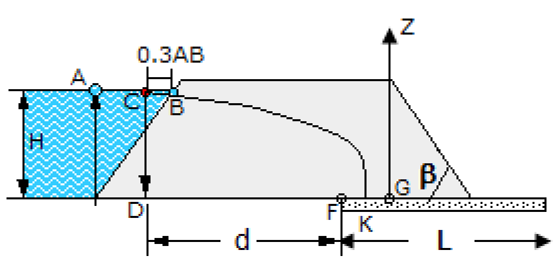


Select the base of the dam and directrix as x and z-axes.

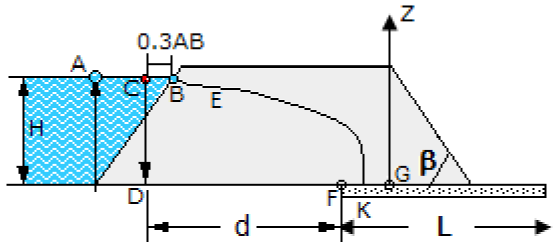
5. Construct basic parabola from

$$\boldsymbol{z = 2\sqrt{p(p + x)}}$$

Choose arbitrary values of x and compute z. Join all the points to get the basic parabola.



6. Upstream Correction: Sketch the section BE such that it is normal to the upstream sloping surface (i.e. equipotential line). BEK is the phreatic line (top flow line)

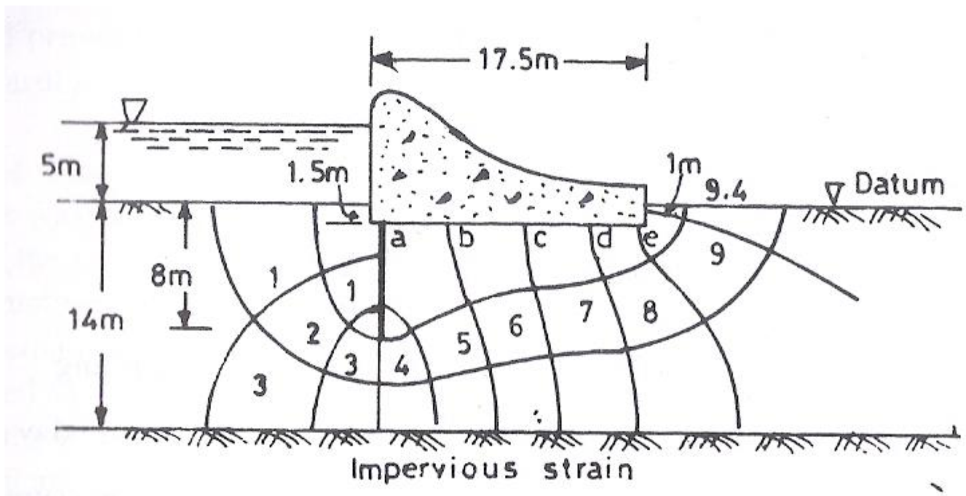


7. Let the distance between focus, F, and directrix, G, be S.
   i.e., $S = 2p$
   The quantity of seepage per unit length of earth dam with toe filter is
   $q = kS = k\ x\ (2\ x\ p) = 2kp$

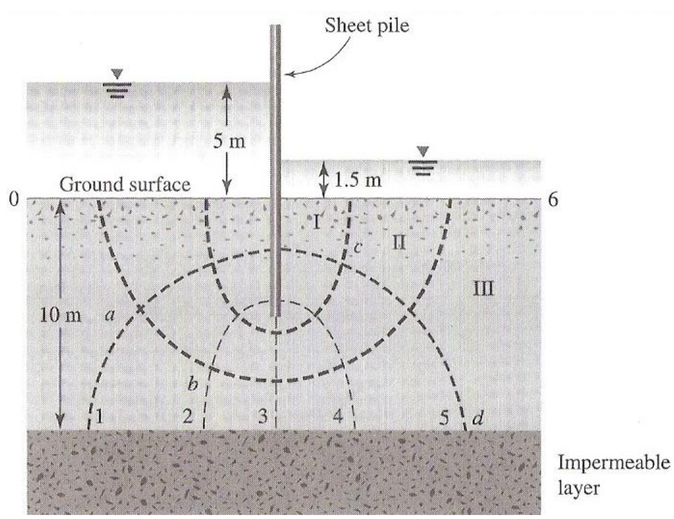
   Where k is the permeability of the earth dam material.

Problems

1. A concrete dam of 17.5 m base width retains 5 m of water. A sheet pile cut off is provided at the upstream end of the base of the dam up to a depth of 8 m. The base of the dam is 1.5 m below ground surface and the pervious foundation extends to a depth of 14 m, below which is an impervious stratum. Draw the flow net and compute the seepage flow below the dam per metre length of dam, if $k = 2 \times 10^{-3}$ cm/s. Compute the uplift pressure along the base of the dam and the exit gradient. Draw the uplift pressure diagram.

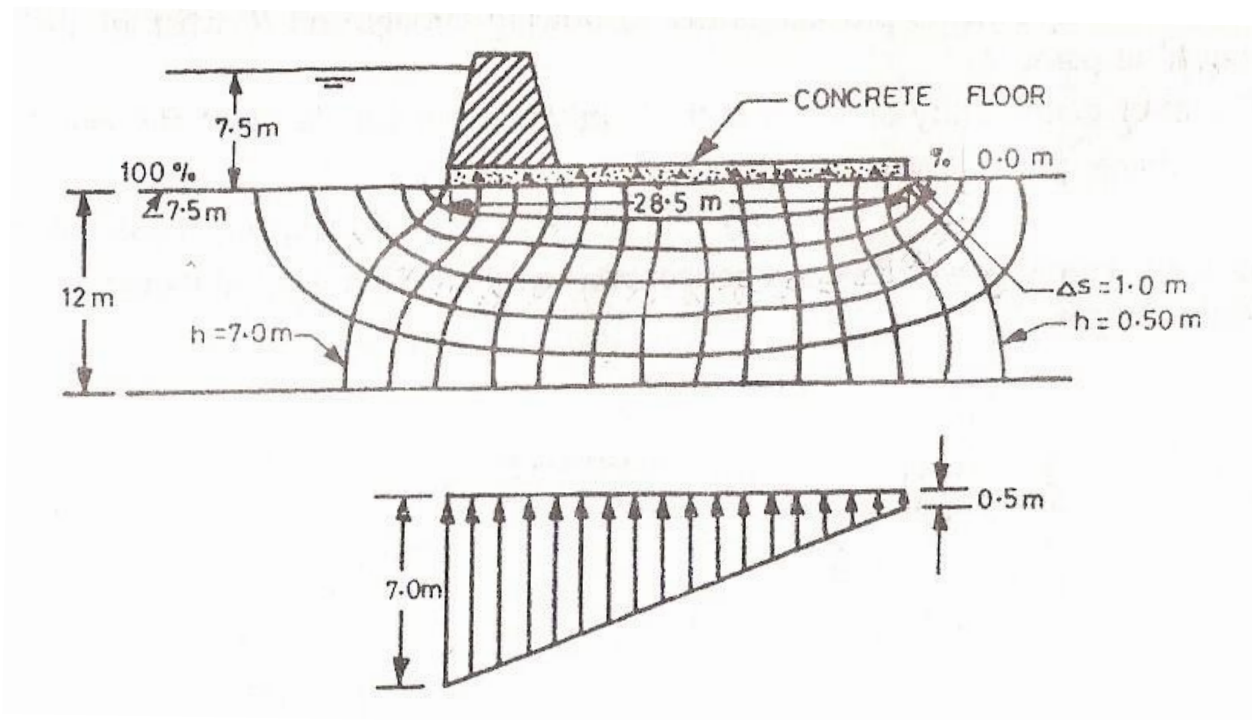


2. A flow net for flow channel around a single row of sheet piles in a permeable s oil layer is shown in figure. If k = 4.2 x 10^{-3} cm/s, determine
   a.) How high (above the ground surface) the water will rise if piezometers are placed at points a, b, c, and d.
   b.) The rate of seepage through flow channel II per unit length (perpendicular to the section shown).



Assignment

1. Determine the uplift pressure on the concrete impervious floor of the weir shown in figure. Also determine the exit gradient.



2. For a homogeneous earth dam of 52 m height and 2 m free board, the flownet has 22 potential drops and 5 flow channels. Calculate the discharge per meter length of the dam, given k = 22 X 10-6 m/sec.

3. A concrete dam 17.5 m base retains water to a level of 11.0 m on the upstream. The water level on the downstream is 2.0 m. The impervious stratum is 10 m below the dam. The coefficient of permeability $k = 10^{-6}$ m/sec. If dam is 50 m long compute total quantity of seepage flow per day below the dam. Also compute seepage pressure at point P, 5 m below the center of the dam.